Assessment Schedule – 2006

Scholarship Statistics and Modelling (93201)

Evidence Statement

QUESTION ONE (8 marks)
Tasks: (a), (b) and (c)
AS Reference: LP 90644

Evidence (a):

Let x = number of boxes of light strawberry yoghurt.
Let y = number of boxes of standard strawberry yoghurt.

Constraints are: $x + y \leq 400$; $y \geq 200$; $y \geq 2x$ and $y \leq \frac{7x}{2}$

We wish to maximise Profit = $px + y$ where $p > 1$
Optimal point is (133, 267) ie 133 boxes of light and 267 boxes of standard.

Evidence (b):

Constraints $y \geq 2x$ is replaced by $y \geq \frac{5x}{2}$ and $y \leq \frac{7x}{2}$ is replaced by $y \leq 3x$

Optimal point becomes (114,286), ie 114 boxes of light and 286 boxes of standard.

Evidence (c):

Let A = event box contains an unsealed pottle.

Let B = event box hasn't a tucked in corner.

Pr (A) = 1/40 and pr (B) = 1/50 so pr ($A \cup B$) = 0.025 + 0.02 – 0.0005 = 0.0445

No. of boxes on average = $400 \times 0.0445 = 17.8$

Or use pr ($x \geq 1$) = 0.02474 with n = 6 and $\pi = 1/240$ for pr (A)

Assumption: Events are independent.

Sufficiency:

(a) Score S if correct.
Score P if constraints are either shown or written as correct.
Score P if answer is slightly out by 1.
Score P if answer is correct but 1, 2 constraints missing.
Y $\geq$ 200 missing still correct answer score P.

(b) Score 0 if correct.
Score P if changed constraints are either shown or written as correct or if answer is slightly out by 1.

(c) Score S if correct.
If assumption missing score P.
If word "independent" is used with nothing else, score N.
If assumption correct with correct process, score P.

QUESTION TWO (8 marks)
Tasks: (a) (i), (a) (ii), (a) (iii) and (b)
AS Reference: 90643, 90646

Evidence (a) (i):

$Z = -2.326$ so $\mu = 150 + 2.326 \times 8 = 168.6$ ml

Evidence (a) (ii):

$$\mu = 155 \pm 1.96 \times \frac{8}{\sqrt{25}}$$

$\mu = 155 \pm 3.136$ or $151.864 < \mu < 158.136$

From (a) 168.6 is outside interval, so machine isn't filling to achieve 1% below 150 ml.

Evidence (a) (iii):

$$n = \left(\frac{z\sigma}{E}\right)^2 = \left(\frac{8 \times 2.576}{1.6}\right)^2 = 166$$

Evidence (b):

Select a sample and measure its fill volume from each machine.
Calculate the mean and standard deviation of the fill volumes from each sample separately.
Construct a 95% confidence interval for the difference in means.
See if 0 lies within interval.
If so then conclude that machines aren't giving differing fill volumes.

Sufficiency:

1. Combine (a) (i) and (a) (iii).
   Score P1 and P2 if respectively correct.
   S overall if both correct or P if only one is correct or N if neither correct.
2. In (a) (ii) score S if correct or if interval is correct score P.
   Score P if comment is consistent with incorrect interval.
3. In (a) (iii) score N if answer <u>not</u> rounded.
4. In (b) score O if correct or score P if only 1st 3 lines are correct.
5. In (b), if no context score P.
6. In (b) if construction statement correct, can assume mean and standard deviation have been calculated.
7. In (a) (iii), $n \geq 166$ or $n > 165$ is correct.
8. Can have carried error from (a) (i) to score S possible in (a) (ii).
9. Units not penalised.

QUESTION THREE (8 marks)
Tasks: (a), (b), (c) and (d)
AS Reference: 90646 and 90642

Evidence (a):
Percentage of defective strawberries removed by visual inspection
$= 100 \times 0.1 \times 0.7 = 7\%$

Evidence (b):
Probability that a rejected strawberry is removed by the scanner
$= \dfrac{0.9}{0.97} = 0.928$

Evidence (c):
Use Poisson approximation to the Binomial.
Now from Poisson tables when pr(x = 0 or 1) = 0.66, $\lambda = 1.2$ where x is the number of defectives.

Hence $\lambda = np$ so $n = \dfrac{1.2}{0.04} = 30$

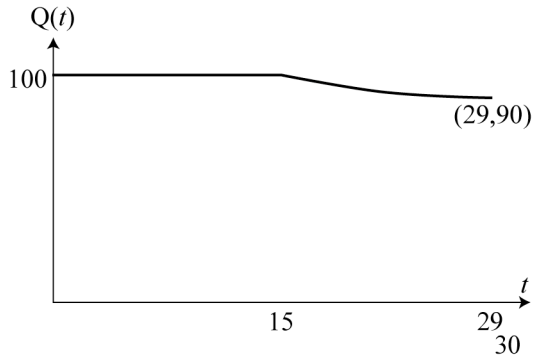
Or equate $0.96^{n} + n(0.04)(0.96)^{n-1} = 0.66$ and solve to give n = 31.

Evidence (d):
Construct a 95% confidence interval for the proportion of defective strawberries based on 1/40 = 0.025 defective.
An interval 0.0 to 7.3% (or lower limit = –2.3%) for the population percentage defective is obtained.
As 4% lies within the interval, we conclude that there is no evidence to suggest that the average percentage defective is now less than 4%.
OR find pr (0 or 1 defective) = 0.323 > 0.05, or pr (1 defective) = 0.3256, so decide that there is insufficient evidence to conclude that the average % defective is less than 4%.

Sufficiency:
1. Combine (a) and (c).
   Score P1 and P2 if respectively correct.
   S overall if both correct or P if only one is correct or N if neither correct.
2. Accept 0.07 in (a) and $\dfrac{90}{27}$ in (b).
3. Score S in (b) otherwise score P if only the 0.97 is correct.
4. Score O in (d) otherwise score P if only the full first line is given.
5. In (b), accept 0.93, if 0.9 score P.
6. Score P in (d) if p = 0.04 is used to compute standard error rather than 0.025.
7. To score O in (d) must have some indication of context and calculation.

QUESTION FOUR (8 marks)
Tasks: (a1), (a2), (a3) and (b)
AS Reference: 90644

Evidence (a1):
Graph should include the points: (0,100), (15,100) and (29, 90) along with the correct shape shown below:



Note: Either a rough sketch showing points or an accurate scale diagram is acceptable.

Evidence (a2):
Form the two equations:
Q (15) = 100 = A exp (15B) and Q (29) = 90 = A exp (29B)
Solve these to get A = 111.9 and B = –0.0075
Or can have A = 112 and B = –0.008.

Evidence (a3):
Start with 95 = 111.907 exp (–0.0075t) and solve for t to get t = 21.8 (21 or 22)
So use-by-date will be Dec 22nd or Dec 23rd 2006.

Evidence (b):
When D = 15, Q drops below 95 at t = 21.8 so if t = 20, 1.8 days earlier, D = 13.2
So we want pr (D < 13.2) = pr (z < –1.083) = 0.5 – 0.3606 = 0.1394 ie 13.9%

Sufficiency:
1. Combine (a1) and (a2)
   Score P1 and P2 if respectively correct.
   S overall if both correct or P if only one is correct or N if neither correct.
2. Score S in (a3) or P if t is correct only (line 1).
3. Score O in (b) or score P if D = 13.2 is obtained (line 1).
4. In (a)(i) all major points need to be shown. Straight line not accepted.
5. In (a) (iii) carried error from (a) (ii) is allowed.
6. In (b) carried error from (a) (iii) is allowed.
7. In (b) can use Q = 95, T = 6.815, μ = 21.31, σ = 1.2 with pr [x < 20]
8. In (b) score O if no % in final answer.

QUESTION FIVE (8 marks)
Tasks: (a1), (a2), (b1) and (b2)
AS Reference: 90644, 90645, 90647 and 90641

Evidence (a1):
Form the equations: $972 = a + 1.2b + 20c$, $872 = a + 1.28b + 24c$ and $722 = a + 1.40b + 30c$.
Solve these to get: $a = 2472 + 40c$ and $b = -50c - 1250$

Evidence (a2):
When $T = 0$, $P = 1.45$ and $N = 0$ we get $c = 20.29$
So when $N = 40$ and $P = 1.45$ we get $T = 812$ ie \$812,000

Evidence (b1):
For February 2007, $t = 39$ so $MA = 89.068 - 0.9667 \times 39 = 51.3667$
The seasonal for SS (Feb) $= (10.5 + 6.2) \div 2 = 8.35$
So Sales of Standard Forecast for Feb 2007 = 51.37 + 8.35 = 59.72 ie \$59,720

Evidence (b2):
Now SR $= 1.8314 - 0.0646 \times 25 = 0.2164$
So Sales of Light Forecast for Feb 2007 = 59720 ÷ 0.2164 = \$275,970

Sufficiency:
1. Combine (b1) and (b2).
   Score P1 and P2 if respectively correct.
   S overall if both correct or P if only one is correct or N if neither correct.
2. Score S in (a1) or P if all equations are correctly formed.
3. Score O in (a2) or P if c value is correctly obtained. If T = 811 or 812 score P
4. The answer to (b2) is conditional on the answer given in (b1) so can be marked correct provided there is no further error.
5. If 59.7 & 275.97 given in (b1) & (b2) as answers score P overall

QUESTION SIX (8 marks)
Tasks: Essay, validity and improvements
AS Reference: 90641, 90645 and 90647

Evidence:

Essay

Over the last three years the sales of standard have fluctuated and moved downwards, dropping by about a third.

Note: Just stating "the sales of standard have fluctuated" isn't good enough to score the point.

In June 2004, the sales of standard reached the lowest point at $39 000.

Sales of standard tend to peak in the summer months and trough in the winter months.

With the absence of the seasonal effect, the moving average graph shows a fairly constant downwards trend between June 2004 and May 2006.

The sales proportion, standard to light, shows a good negative correlation with marketing costs.

There is an outlier at (14.5,1.8) where sales of standard exceeded that of light yoghurt by 80%, which wasn't true of the sales ratio in general where this ratio was steadily dropping while marketing costs were increasing.

Validity

The forecast for the sales of standard, $812,000, over the year is suspect as it's based on too few points, it assumes no changes in the values of *N* and *P* and other competition isn't taken into account.

The forecast for sales of standard, $59 720, is reasonable as February 2007 isn't too far into the future however the seasonal effect for February has been estimated only over two years.

The forecast for the sales of light, $275 920, is unrealistic due to its value and the fact that a marketing cost of $25 000 lies outside the range of the data and quite clearly the fitted line no longer applies.

Improvements

Obtain more data, say for each month, on number of sales outlets, market share and price movements so that other models can be created. Note: Just stating "obtain more data" isn't good enough to score the point.

Look at fitting a model to a time series plot giving the sales of light over each month.

Don't attempt to predict too far away from the range of data used to create the models.

Sufficiency:

Essay – Score S if 3 points made or P if 1 or 2 points made (use √e) otherwise N.

Validity – Score S if 2 points made or P if 1 point made (use √v) otherwise N.

Improvements – Score O if 2 points made or P if 1 point made (use √i) otherwise N.

Points need to be fully made for credit.

No penalty for wrong points.

Scoring of Questions

Each question is scored as:

8: 2S + O
7: 2S + P or S + O + P
6: 2S or O or O + P or O + 2P or O + S
5: S + P or S + 2P
4: S
3: 3P
2: 2P
1: P
0: N or 2N or 3N

Scores are added together over the six questions to give a total score of between 0 and 48 inclusive.